Fluid Mechanics
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Lecture No. – 08
Conservation of Mass

Welcome all of you to this fluid mechanics course. Today, we are going to plan about conversion of mass. As you could remember it, then, in the last class we discussed about Reynolds transport theorem. So, the same Reynolds transport theorem will be used to derive mass conservation equation which is an important equation for any fluid flow problems.

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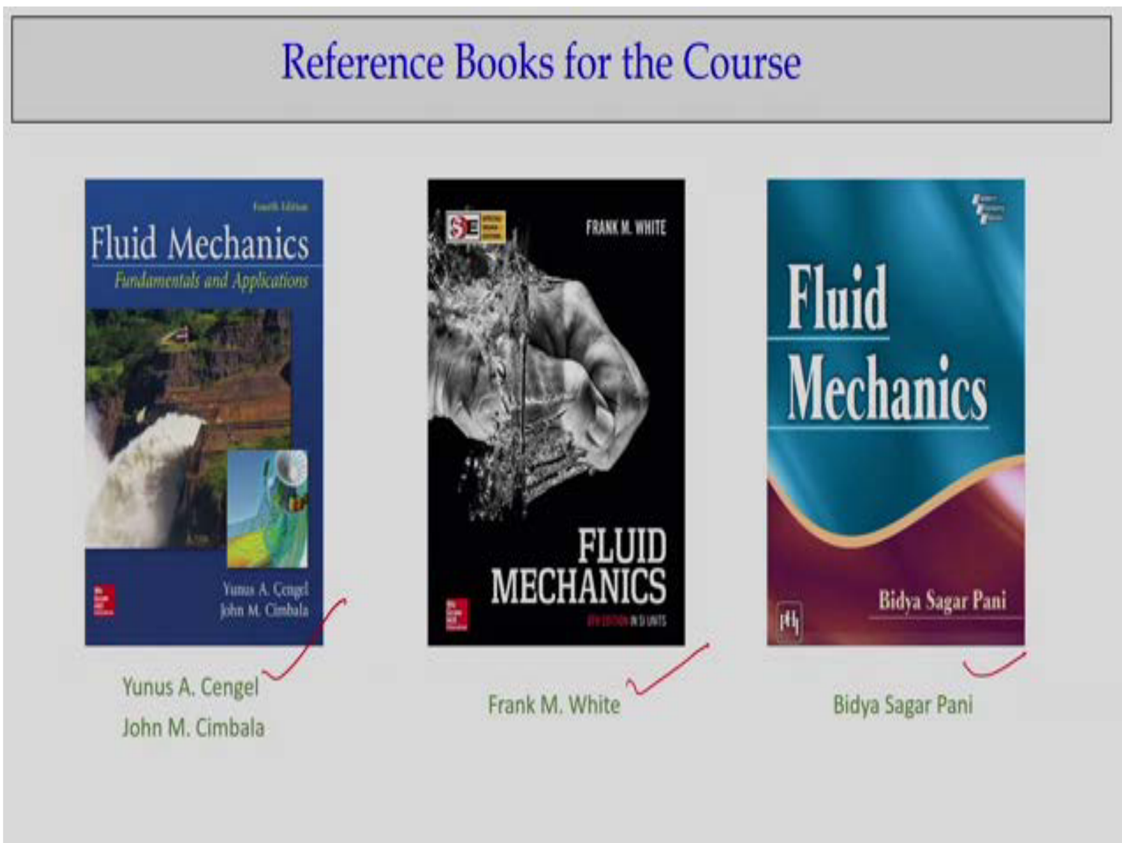

Now, again, I can repeat that you can have this reference books. Cengel and Cimbala, Fluid Mechanics, Fundamentals and Applications. Then, F.M. White, Fluid Mechanics, mostly the derivations and partly we have been following this book. And this Fluid Mechanics by Prof. Bidya Sagar Pani.

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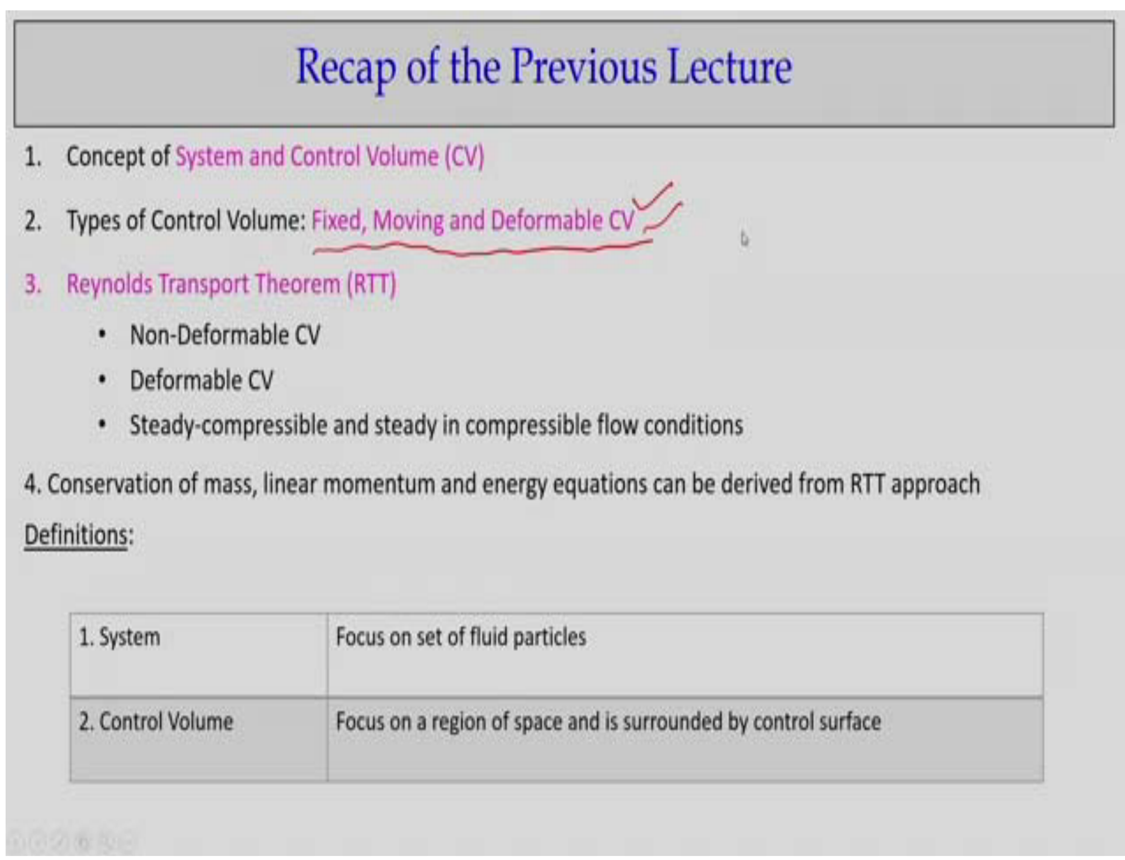

Let us come back to what we learned in the last class. If you remember it, I discussed very thoroughly what is the difference between systems and the control volume. Mostly in fluid mechanics we follow the control volume aspect. That is the reason we need a relationship between the system and the control volume. The Reynolds transport theorems establish the relationship of conversions of mass from system to control volume.

So, that is the strength of the Reynolds transport theorems and, as already I discussed, you can have three type of control volumes, fixed control volume, that means with reference to the space it remains at the same location. As the time goes, it does not move it. That is what fixed control volume is. It has definite control surfaces. The other one is moving control volume, like the ship movement in a sea or the river. The ship and the adjacent fluid can be considered as moving control volume.

And we solved many of the problems with moving control volume, considering this control volume is moving with a particular velocity. Not only that, the shape of the control volume can change. That was the deformable control volume. The shape of the control volume can change with respect to time, then we call it deformable control volume. So, there are three types of control volume, fixed control volume, moving control volume, and deformable control volume.

So, in deformable control volumes, the shape of the control volume changes with time. In case of moving control volume, the control volume moves with a particular velocity. That is what is moving control volume. And fixed control volume, as you can see it remains stationary at that space regions. Today, we will talk about how we can derive the conservation of mass.

As I said earlier, when you have the control volume, through the control surface there is, mass influx coming into the control volume, going out from the control volume. Similar way, momentum flux comes into the control volume, also goes through the other surface as momentum flux going out from that. Similar way, we can think this energy flux comes into the control volume and goes out of this thing.

So, that way the conservation of mass, linear momentum, and energy equations, that is what we will discuss in more detail. Today, I will focus on mass conservations only and that is the point we will discuss.

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So, as I have given very briefly Reynolds transport theorems and moving and deformable control volume, conservation of mass we will derive for simple case to complex problems, how we can write conservation of mass. Then, we will talk about some examples, real life examples we will take and how the appropriate choosing of control volume and the control surface can solve many of our problems.

That is what I will demonstrate, and the chosen examples we will tell to you, and end of the day, I will summarise whatever the lecture component today I will deliver.

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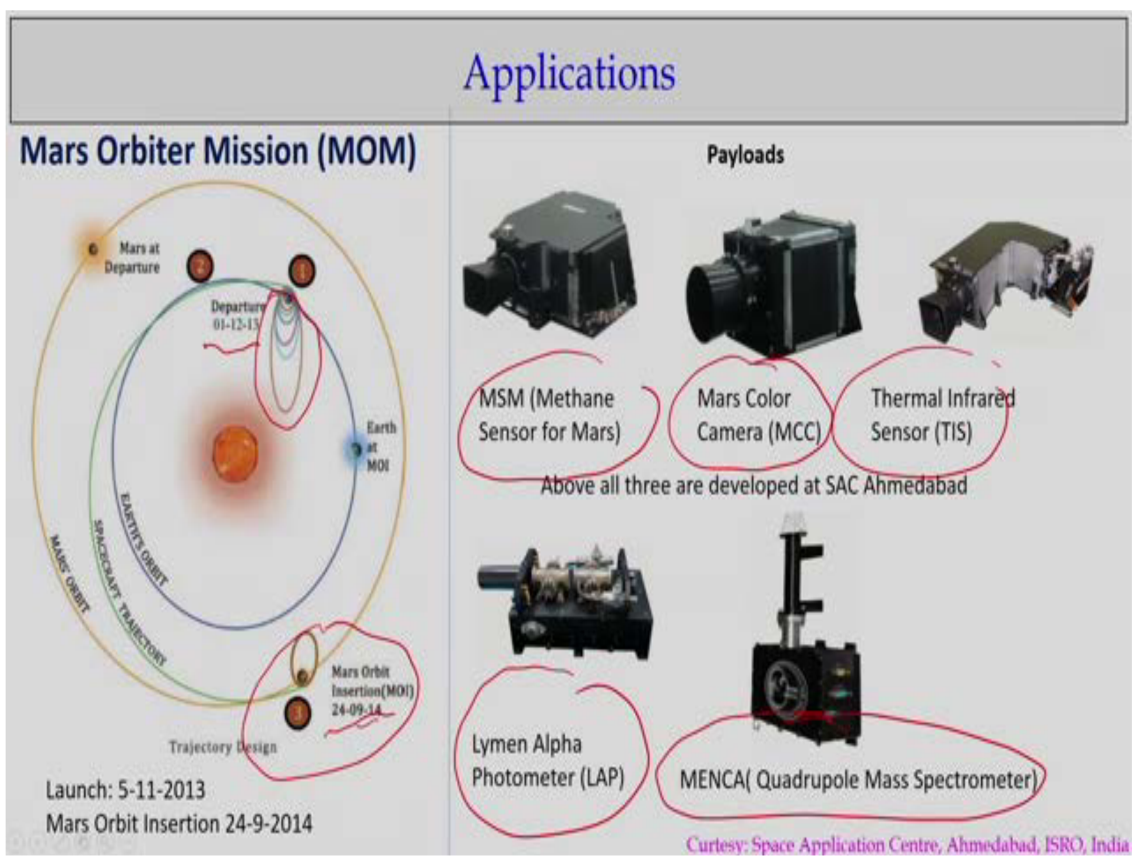

Now, let us go to very interesting applications. As you know it Indian Space Research Organisation launched the satellite which is called MOM programmes, that means Mars Orbiter Mission programme. But if you look at this, it is the fluid flow problem. If we look at the trajectory which is started from the earth, and if you look at the trajectory part here, how this trajectory goes on changing, which was launched on December 1, 2013, okay?

It is to reach the Mars orbit September 24, 2014. If you look that all satellites moving in the atmosphere, the space, so accurately, considering your drag force and all the things, so accurately the trajectory was designed to reach Mars orbit after almost more than one and half years. So, the fluid mechanics knowledge what we have today we can design the trajectory from earth to the Mars orbit. What I am trying to say is that fluid mechanics today is expanded in a very, very high applications way.

So, what we are to study in this course is introductory levels. And, if you have interest, then really we can solve these problems which we have done for Mars orbiter missions, launching the satellites from earth to reach up to Mars orbiter locations. And you look at the payload or the sensors, whatever was there in the Mars orbiter missions, starting for methane sensors, the colour cameras, infrared sensors, or alpha photometers, and quadrupole mass spectrometer.

I am not going to discuss each sensor what it does, okay? If you are interested, you can get a lot of literatures available in the internet but what I am trying to say is that India has capability or Indian scientists have the capability to design a satellite tracking systems with the sensors, starting with launching from the earth's surface and it can be reach up to the Mars orbiter.

Not only that, these sensor, whatever design it is, that also can get the data about Mars and that is what is the target, that is what succeed. So, all these knowledge of fluid mechanics, the advanced mathematics, all this help us to reach this mission's programmes, what is done by Indian Space Research Organization. With this note, let us talk about the fluid mechanics part.

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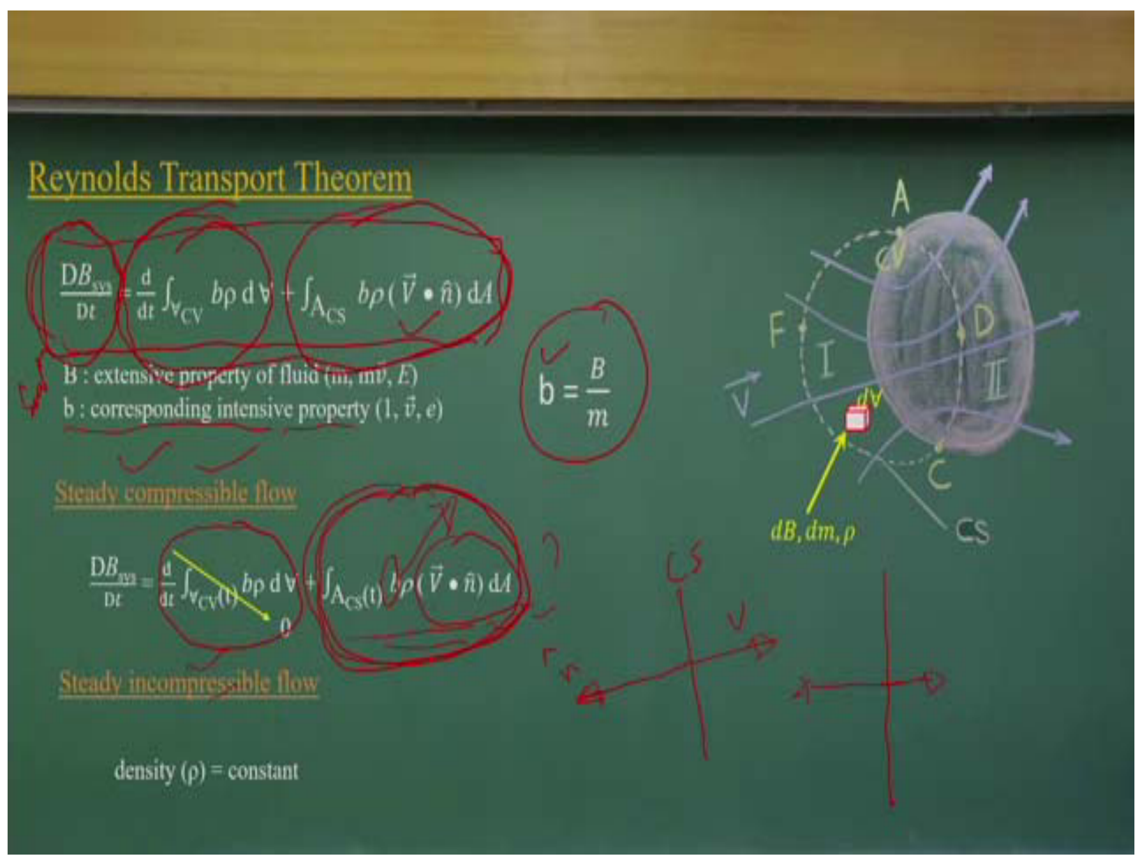

Coming back to the Reynolds transport theorems, as I derived in the last class, one is the system level equations and the other is the control volume level equations.

$$\frac{\mathrm{D}B_{\mathrm{sys}}}{\mathrm{D}t} = d/dt \int_{\forall_{CV}} b\rho \,\mathrm{d}\forall \; + \int_{A_{CS}} b\rho\,(\vec{V} \bullet \hat{n})\, dA$$

B : extensive property of fluid (m, m$\vec{v}$, E)

b : corresponding intensive property (1, $\vec{v}$, e)

b = $\frac{B}{m}$

This is intensive property. Extensive properties are mass, momentum, and energy.

Intensive properties are unit value for the mass, for the momentum, the velocity vector for energy is also e, the specific energy, energy for unit mass. So, if you look at this equation, let us look it. This is the system level equations. This is what is at the control volume level. So, one talks about how this particular mass or momentum flux is crossing through the control surface. The net outflow is going through this control surface.

That is what it is and this is what it is talking about. The change of the extensive properties within the control volume with respect to time, one is with respect to control surface, one other is with respect to storage within the control panel, how that extended property is changing with time, that is related, the system and the control volume level. Now, let us have two simplifications or assumptions which is quite valid for the fluid flow problems for most of the times. That is steady and compressible flow.

$$\frac{DB_{sys}}{Dt} = d/dt \underbrace{\int_{\forall_{CV}} b\rho \, d\forall}_{0} + \int_{A_{CS}} b\rho \, (\vec{V} \bullet \hat{n}) \, dA$$

That means the density vary but it is steady. So, once it is steady, if you look at this term, it becomes 0. Only what remains is the surface integration part which is quite easy now. If you take a steady problem or if you can visualize the problems, it can be solved as a steady problem. Then you need to go for complicated volume integrals and partial derivative or total derivative of these things.

You can focus on only the control surface and over this control surface we can have surface integrals and we can equate this equation. But when steady and incomprehensible, that means this density becomes a constant quantity. So, density can come out from this surface integral. That means only the b and velocity, the scalar products, the velocity and unit vectors, that part will do the scalar part and surface integrals.

Now, if you look if I have the velocity vectors, okay? Let us consider I have a control surface like this. If the n vector is like this, the perpendicular vector to the control surface is like that, if I consider one simple case, the velocity is either parallel or opposite, if the same direction or opposite direction, then my dot product is very easy, either v positive or v negative values.

What it indicates is that if you consider the control surface in such a way that your normal vectors and your velocity vectors either will have the same direction or opposite direction of that. So, if that is the condition, your scalar products will give it v positive or v negative depending upon $\cos\theta$, 0° or 180° degrees. So, it is very easy to choose appropriate control surface and find out what could be appropriate normal vector to that control surface.

If I know that, that means that should be aligned with my velocity vectors. So, once I know these velocity vectors I can have a approach, make my control surface in such a way it should be perpendicular to that or the unit vectors or perpendicular vectors that should equally have the same line or the opposite direction. The tricks what we follow is to simplify the problems instead of doing the scalar product.

We can do it, it is not a difficult task, but we can simplify that by taking appropriate control surface. That is to be remembered. Appropriate control surface taken such a way that this scalar product can be simplified. So, once it is simplified, only we need to do surface integrals of the velocity and the dA and the b value. Now the problem is quite simple for steady incomprehensible flow.

density (ρ) = constant

Mostly my lectures will talk about this part only.

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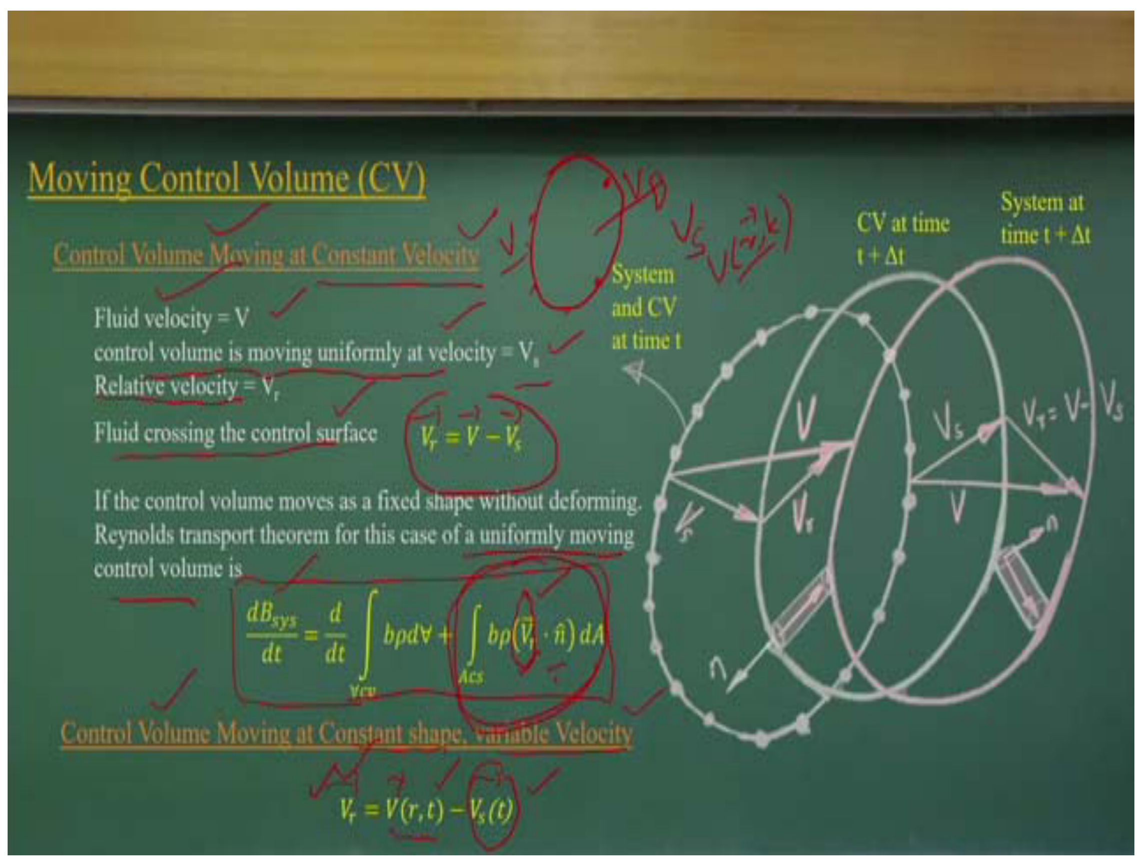

Now coming to a very interesting **(()) (13:15)** if I consider moving control volume like a ship which is moving with a velocity V. So, then, the concept will be the same. Only here we will talk about the relative velocity component. That means there will be fluid velocity V and control volume is moving with a velocity, let it be V_s. So, we need to know it, what will be the relative velocity V_r crossing through this control surface.

Fluid velocity = V

control volume is moving uniformly at velocity = V_s

Relative velocity = V_r

Fluid crossing the control surface

$$V_r = V - V_s$$

For example, if the V is 5 meter per second and V_s is moving in the same direction with 3 meter per second, the relative velocity will be 2 meter per second. So, that way find out what will be the relative velocities coming into a moving control volume. That is what generally you do in any physics problem. The same way where the control volume is moving with velocity V.

That means the control volume is moving with a velocity V s and the fluid velocity is V. So, we have the relative velocity which is $V_r = V - V_s$. You can represent it as velocity vector form. Then, this can come as vector form which will be complicated or you make it only one directional velocity. So, that way you can make $V - V_s$ or we can use a vector form. So, if that is the condition, if it is moving at this, the Reynolds transport theorem for uniform moving control volume will be the same. Here, we are using this V_r, the relative velocity.

So, this will be the vector form, the subtraction of V and V_s. We are using this relative velocity component and dA, that is what is coming from this part. Please take care of these things that when you consider the control volume moving with a constant velocity we use the relative velocity things. And the relative velocity, if it is one dimensional velocity field, it is very easy, but if not, you do this vector subtraction and that vector subtraction can be used to find out the scalar product of this and that is what is represented here.

Your Reynolds transport theorem has just modified it considering instead of the V, the fluid velocity, we use the relative velocity here, where V stands for velocity of the control volume. Now, let us consider another case. Like the control volume moving with a constant shape with a variable velocity, okay? So, that means what we are talking about is that the control volume is moving, it is not a constant velocity it is moving with.
At each point when it is moving it has the velocity which is the function of position vector and the time, okay, this position vectors with time, if the velocity of the flow is coming into this system is positions vector t and V is moving with a velocity t, varies with time. That means, you are not a fixed velocity, varying the velocity. Like you are accelerating control volume.
If the control volume moves as a fixed shape without deforming. Reynolds transport theorem for this case of a uniformly moving control volume is

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{\forall cv} b\rho d\forall + \int_{Acs} b\rho\left(\vec{V}_r \cdot \hat{n}\right) dA$$

Initially it may have less velocity, as you go the velocity is changing, okay, and V_s is changing, or you are deaccelerating, V_s is changing, and velocity vector position you know it what will that be. In similar way, you can get the relative velocity. Again, vector subtraction of these two velocity fields you can get the V_r value. Substitute the V in Reynolds transport theorem, again you will get the equations for the control volume moving at constant shape with a variable velocity.

$V_r = V(r,t) - V_s(t)$

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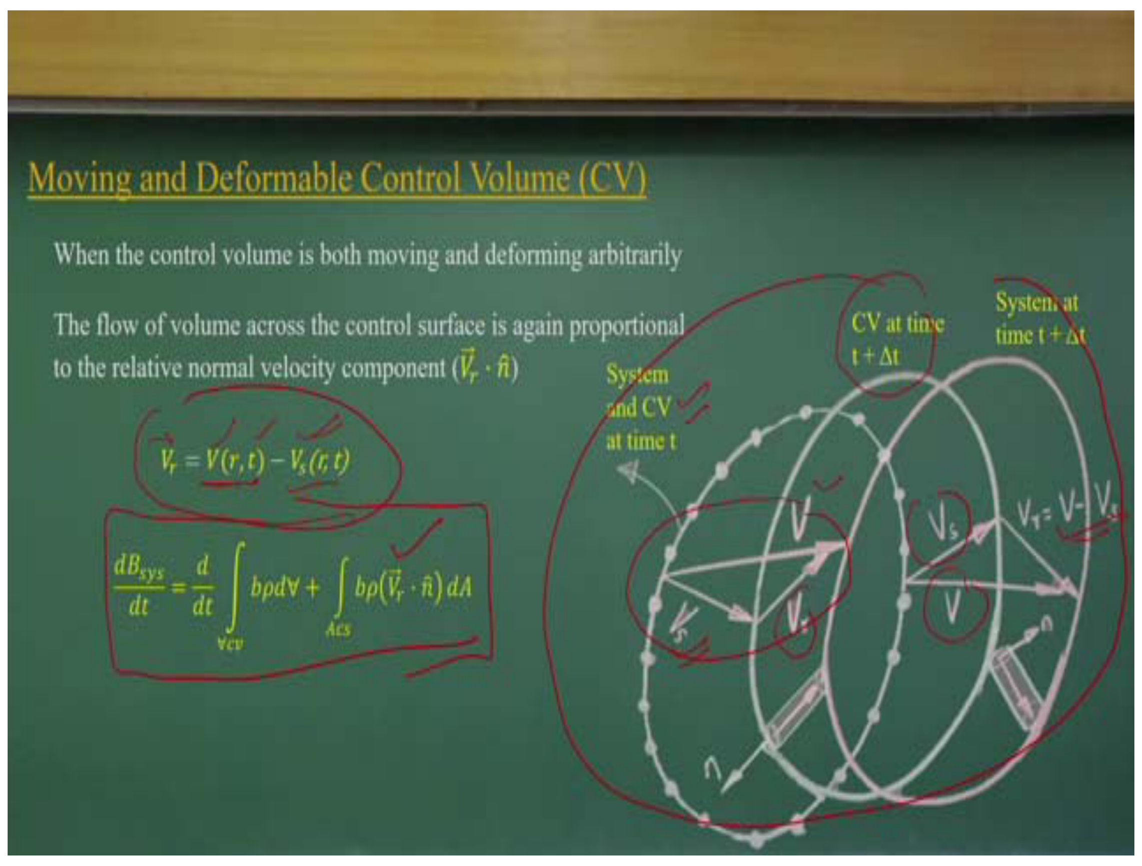

But in case you have a deformable control volume and also moving control volume, all the things you have are complicated now. That means you have V which varies with position as well as time, okay? So, like, for example, here it is given system control volume t equal to time, $t + \Delta t$ time. So, your V_s and V, fluid vectors have a function of position and the time. Similarly, because it is deformable control volume we consider your V also varies with position and time. So, it is r and t dependent, so position and time dependent.

$V_r = V(r,t) - V_s(r,\ t)$

And that vectors difference, if you see this graphically, the resultant velocity V_s and V and V_r is relative velocity. Similar way, for this surface, you know the V_s, you know the V, you know the V_r. So, this is what is represented for you, how the moving and deformable control volumes as it is moving, how the velocity fields changes, how the velocity with respect to the control volume changes from each point to point. Each point to point you can have the difference because it is depending on position vectors and also the time.

So, if you look at the similar equations only, the V_r, we are deriving this relative velocity vectors which is different for different cases like moving control volume, moving control volume with deformable which will be coming like this. So, again, I am going to repeat it, please do not be afraid of the surface integrals and the volume integral things because most in of the engineering problems we simplify these two integrality in such a way that we need not do the integration.

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{\forall cv} b\rho d\forall + \int_{Acs} b\rho(\vec{V}_r \cdot \hat{n})\, dA$$

We try to have just area multiplication or constant velocity **(()) (19:41)** concept we use it. But for any complex problems where the geometry of the control volume can have complex natures, at that time we need to surface integral and volume integrals. But again I am going to repeat, do not be afraid. We have a lot of mathematical softwares which does the surface and volume integrals for us.

So, how to do the integrations for the surface level or in a volume control level, it is not a difficult task at the present area, but I will simplify the problems which is coming for your competitive exams.

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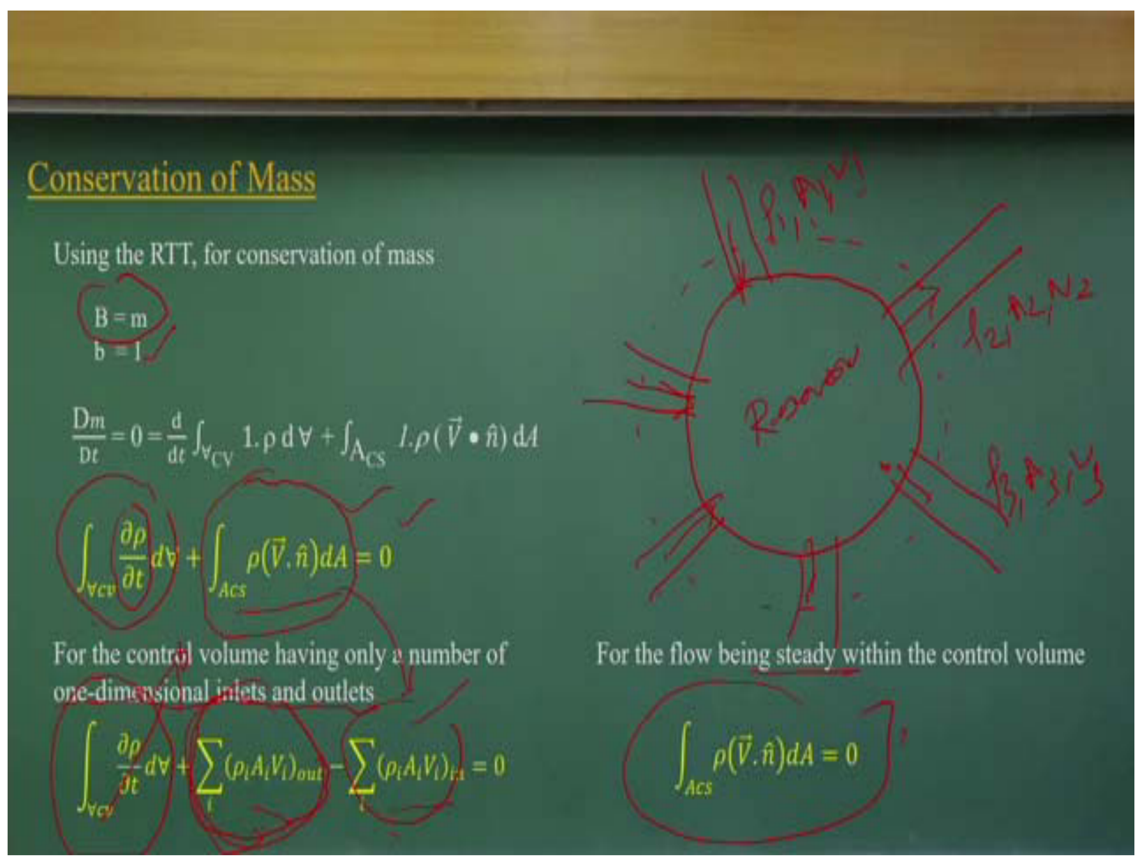

Now, let us come to the, as I said it, we need to have three equations. One is conservation of mass, conservation of linear momentum or the angular momentum. And third is, as you know it, conservation of energy. So, most of the times you remember whenever you solve the fluid mechanics problems we have three helping hands, it is not two, it is three helping hands. One is conservation of mass, the other is momentum, and another is energy.

But the problem is that many of the times we solve only the pressure and the velocity, okay? Many of the problems we are going to solve in this introductory fluid mechanics. Only need to solve for the pressure and the velocity. So, you may not need three equations. You just need two equations. In that two equations first and foremost requirement is always mass conservation will be there.

Whatever the problems you solve it, the first thing is that mass conservation will be there. Only the option left with us is whether you have to consider conservation of momentum or the energy, thus the difference, because we have only two solutions we are looking at, we have three equations. That is why it is quite interesting in fluid mechanics, sometimes we use the linear momentum equation, angular momentum equation, or the energy equations depending upon the problems. So, how easy to solve the problems.

That is what you try to get. So, let us come in to the mass conservation equation which is very easy equation to be solve and the conservation of mass. At the Reynolds transport levels if I put

B = m

b = 1

and no doubt that $\frac{\mathrm{d}}{\mathrm{d}t}$ or $\frac{\mathrm{D}m}{\mathrm{D}t}$ for any systems, that is 0, unless otherwise you have nuclear energy or additional energies coming to the system, what in general you do not consider in this case.

No chemical process, only we are talking about the process where the additional mass is not coming to that or mass is decaying from the system. If you look at the things, substituting b equal to 1. Now simplify the problems.

$\frac{\mathrm{D}m}{\mathrm{D}t} = 0 = \frac{\mathrm{d}}{\mathrm{d}t} \int_{\forall \mathrm{cv}} 1.\rho \, \mathrm{d}\forall + \int_{\mathrm{Acs}} 1.\rho \, (\vec{V} \bullet \hat{n}) \, \mathrm{d}A$

If you substitute b equal to 1, if you look that the density will be a function which we can put, and you have this part. So, the volume integrals and surface integrals here.

$$\int_{\forall cv} \frac{\partial \rho}{\partial t} d\forall + \int_{Acs} \rho(\vec{V}.\hat{n}) dA = 0$$

And this is a very interesting equation. Now, if you look that, this is talking about the net outflow of the mass through this control surface. That means, if I consider a control surface, there will the regions, which are the inflow reflow and there will regions which will be the outflow. And there will be regions where there is no exchange of mass, no flow region. So, that means what you are looking at? You are looking at inflow mass and outflow mass.

$$\int_{\forall cv} \frac{\partial \rho}{\partial t} d\forall + \sum_i (\rho_i A_i V_i)_{out} - \sum_i (\rho_i A_i V_i)_{in} = 0$$

The net change of the mass in this control volume should be equal to the change of mass storage within the control volume. That is what the concept is you are look. If I put as a unit level, the density Kg/m^3. Multiple with the volume which is the m^3. So, what it indicates for me is that Kg per unit time. So, this is how much of Kg of mass storage is happening per unit time. That is what we are getting from this within the control volume.

$$\int_{Acs} \rho(\vec{V}.\hat{n})dA = 0$$

This is the net outflow of the mass. If you look at the V into d A into the d A into the rho. V is the velocity, if you just look at the dimensions, just I am putting the dimension part which is easy to remember or whenever you write the equation, by mistake something, always check the dimension of the problem, okay? The dimension of any equation component should be the same. For example, the density, again I can say that it is Kg/m^3.

Velocity is m/s into area is m^2. So, finally it is coming Kg/s. So, these two equation component is Kg/s. This is mass flux. The net outflow of the mass within this control volume will be the net change in the mass within the control volume. How much of crossing through it, crossing things and the net flux. So, that is what is the mass thing.

But many of the times we simplify the problems. We make it one dimensional inlet and outlet problem. What is that? Let us have the problem of the pipe network connecting to one reservoir, okay? Like you have a big reservoir here. There are the pipes that are connected. A series of pipes are connected. Some are inflow, some are outflow, okay. If this is the reservoir, let this be from the same liquid.

So, you have ρ_1, A_1 and V_1. Here could be ρ_2, A_2, V_2; ρ_3, A_3, V_3. Like this we can have some are inflow, some are outflow if you consider one dimensional flow. Second dimensions we are not talking or third dimensions we are not talking. We are assuming that flow is one direction. I know the average velocity of this things. If it is that, then very easy problem now.

I can, instead of doing this surface integrals, okay, if I take this is my control volume, so I can just get the mass flux per unit time, it will be the density multiplications of area into the average velocity, ρ_1 A_1 V_1, out, in. So, there will be in and there will be the out. So, here, we remember out is a positive, that is because of scalar product, okay. That will be the positive sign here and you will have a negative sign.

So, if you look very tactically, if the flow is one dimensional nature if we present it, in terms of average velocity V_1, V_2, and V_3, ρ_1, ρ_2, ρ_3, and A_1, A_2, A_3, as flow is only one direction, instead of doing the surface integrals we make it a simple summation of each inflow and outflow. That net out mass influx is equal to change in the mass storage within the control volume.

But it is steady, as you know it, this becomes 0. So, you have very simple problems. Only the summations is what you will do for inflow and outflow. So, that means what happens is that, like, for example, your bank account, okay. If the money what you put in and what you spend they are equal, your bank account balance will remain the same. That is what happens in fluid flow problems.

If the problem is steady, there is no change in the storage within the control volume. What it indicates is that net outflux of this control volume becomes zero. What it says is that the sum of influx of mass is equal to some of mass outflux going through the control surface. That is the examples I have given. Some are influx, some are outflux. You just find out at the mathematical level this or as a simple summation for one-dimensional flow.

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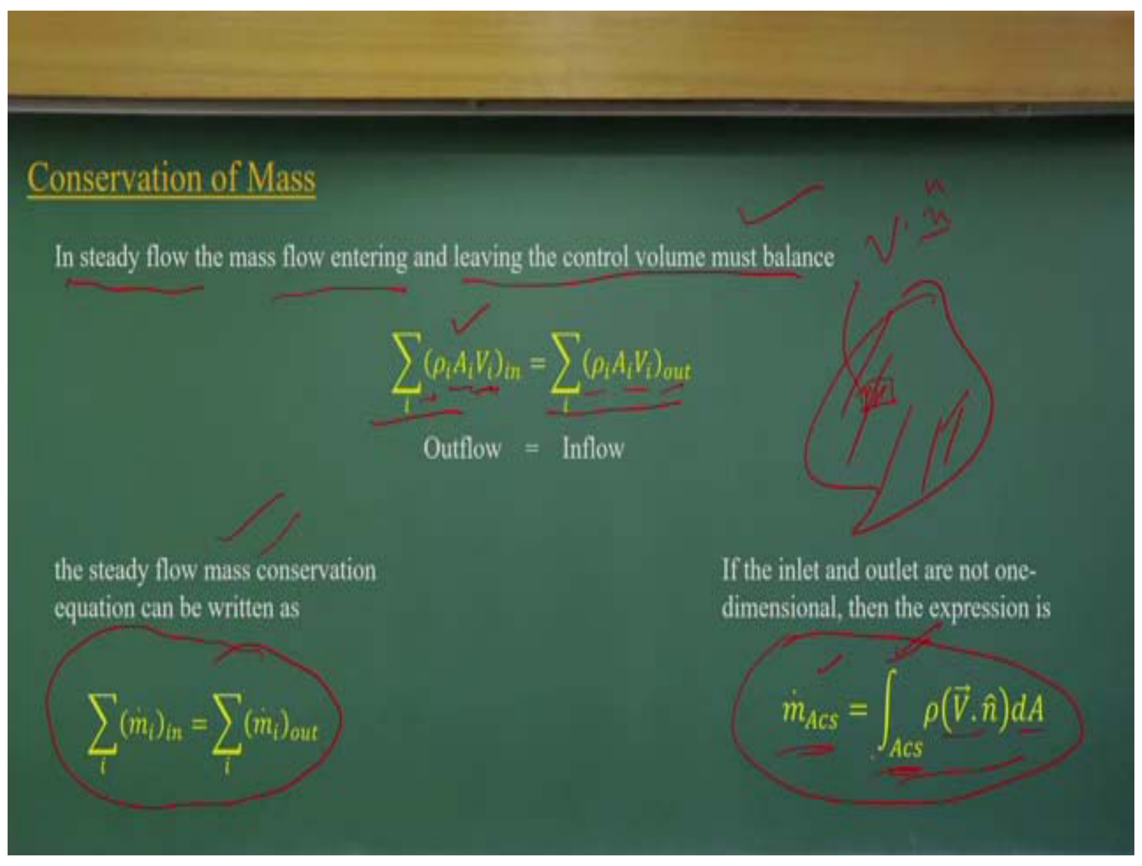

That is what again I am going to repeat it to tell you that further steady flow, mass flow entering the leveling control must be balanced. It is too packed, there is nothing else. Any system, if it is steady, it does not change with time, with respect to time. That means, what it indicates is that there is no change in the storage, mass flow, there is no change in the mass storage within this control volume. So, inflow should be equal to the outflow.

That is what you can do the summations of the number of inflow, the number of the outflow. We should know density, we should know the area of the flow, we should know the average velocity. So, similar way, we can have in and out, we can do the summation of that. If you have same steady flow mass conservation mass level or this one, the same concept, only here we are putting it.

$$\sum_i (\rho_i A_i V_i)_{in} = \sum_i (\rho_i A_i V_i)_{out}$$

And here I am talking about one-dimensional flow. If control surface and velocity vectors they are not aligning, they are not parallel, then you need to do scalar products, then do surface integrals to find out what could be the mass flow going through that control surface. These are surface integration, the scalar product. Then, you do the integrations if the density is also a function of the space and the time, then you will have more difficulty.

the steady flow mass conservation equation can be written as

$$\sum_i (\dot{m}_i)_{in} = \sum_i (\dot{m}_i)_{out}$$

If the inlet and outlet are not one-dimensional, then the expression is

$$\dot{m}_{Acs} = \int_{Acs} \rho(\vec{V}.\hat{n})dA$$

Otherwise, if the density is constant you can take it out, only this velocity scalar product and the d A we can do a surface integral to compute what will be the mass flux. That means if you consider it is very complex control surface as you like it to make it complex things. Then, there will be no problems, you can solve the fluid problems. You need to have a leverage or you need to use high-end mathematic tools.

In today's world what is available like MATLAB, Mathematica and all to integrate it. Considering that for each reach, the velocity varies and each point also your end vectors vary. Do the scalar product, integrate with d A and get it, what will be there. That is possible, but as I say, various ways, because tools are available, only you have to use that tools. For example, for the academic point of view we will not go to doing surface integrals in exams or any competitive.

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